About the end of the electron spectrum in five–lepton μ^+ decay

N.P.Merenkov and O.N. Shekhovtsova

Kharkov Institute of Physics and Technology
61108, Akademicheskaya 1, Kharkov, Ukraine

Abstract

The spectrum of very fast electrons in five–lepton decay $\mu^+ \to e^- e^+ e^+ \nu \bar{\nu}$, that is the main background decay at the study of the muonium–antimuonium conversion in vacuum, is considered. The essential decrease of the spectral distribution is demonstrated when the energy of one positron in this decay is small. Some arguments for such deacrease for arbitrary positron energies are given.

PACS 12.20.Ds

1. The search of deviation from Standard Model (SM) and the probe physics beyond it are the main goals of the current and future experiments in the elementary particle physics. In this connection, the study of the spontaneous conversion of muonium ($\mu^+ e^-$) into antimuonium ($\mu^- e^+$) is of the great interest nowadays because in this process the additive lepton family (generation) number would violate by two units. There are different theoretical models beyond SM where such violation is permitted [1–5].

In recent experiments [6,7], the observation of muonium atom in vacuum have been used to investigate the conversion process. The conversion event would manifest itself by the registration of the fast electron that appears due to standard decay of μ^- from antimuonium atom

$$e^- \mu^+ \to e^+ \mu^- \to e^+ + e^- + \nu + \bar{\nu} \ . \tag{1}$$

The main electrodynamical background for such events in vacuum arises due to five–lepton decay of μ^+ in muonium

$$\mu^+(p) \to e^-(p_3) + e^+(p_1) + e^+(p_2) + \nu\bar{\nu}(q) \ . \tag{2}$$

It is well known that in the case when the energies of both positrons in decay (2) are small enough, the energy distribution of the electrons is strong suppressed at the end of their spectrum. The form of the spectrum at these conditions is [8,9]

$$\frac{d\Gamma}{\Gamma_0 \, dy} = \frac{\alpha^2}{\pi^2}(1-y)^2 F(L,l) \ , \quad L = \ln\frac{M^2}{m^2} \ , \quad l = \ln(1-y) \ , \quad \Gamma_0 = \frac{G^2 M^5}{192\pi^3} \ , \tag{3}$$

$$1 - y \ll 1, \quad x_1 + x_2 < 1 - y, \quad y = \frac{2\varepsilon_-}{M} \ , \quad x_{1,2} = \frac{2\varepsilon_{1,2}}{M} \ ,$$

where $M(m)$ is the muon (electron) mass, $\varepsilon_-,\ \varepsilon_{1,2}$ are energies of the electron and positrons, respectively, and $F(L,l)$ is the known function (see below). The additional smalness $(1-y)^2$ arises because the trivial phase space factor of positron energy fractions: $\Delta x_1 \Delta x_2 \approx (1-y)^2$.

Just this additional smallness of the probability of the background decay (2) makes the selection events near the end–point of the electron spectrum in the process (1) very attractive to observe the muonium–antimuonium conversion.

In Ref. [6] it is suggested that in the general case, where the positron energy fractions x_1 and x_2 can be arbitrary possible, the form (3) of the differential width breaks down and the small factor $(1-y)^2$ disappears. In present work we want to explain that this factor remains independent on values of x_1 and x_2. The physical reason for this assertion is the decrease of the angular phase space of the positron with the large energy. Namely, if the electron in decay (2) carries away the energy fraction y such that $(1-y) \ll 1$ and positron with 4–momentum $p_1\ \ (p_2)$ has the energy fraction $x_1\ \ (x_2) \gg (1-y)$ then it fly just in the opposite direction respect to the energetic electron one and $\Delta c_1 \Delta c_2 \sim (1-y)$, where $c_{1,2} = \cos\widehat{\vec{p}_{1,2}\vec{p}_3}$. We calculate analytically the function $F(L,l)$ for the case when the energy of one positron is smaller than $1-y$ and the energy of the other one is arbitrary. When calculating we neglect with terms of the order of $(1-y)$ in this function.

2. In our calculations we do not take into account the indentity of positrons, because the coresponding effects consist no more than five per cent in a general case (in accordance with Monte Carlo calculations [10]) and trend to decrease in the considered here case when $1-y \ll 1$ [9]. Besides, we use the relativistic approximation and neglect the electron mass always where it is possible. We start from the differential width of the five–lepton decay (2) in the following form

$$\frac{d\Gamma}{\Gamma_0 d\,y} = \frac{\alpha^2}{4\pi^2} R x_1 x_2 dx_1 dx_2 dc_1 \frac{d\Omega_2}{2\pi}\ , \tag{4}$$

where in the used approximation the quantity R can be written as a contraction of two tensors [9]

$$R = [a_1 \widetilde{g}_{\mu\nu} + 2a_2 \tilde{p}_{2\mu}\tilde{p}_{2\nu} + 2a_3 \tilde{p}_\mu \tilde{p}_\nu + 2a_4 (\tilde{p}\tilde{p}_2)_{\mu\nu}][(p_1p_3)_{\mu\nu} - \frac{k^2}{2} g_{\mu\nu}]\ , \tag{5}$$

$$k^2 = (p_1+p_3)^2\ ,\quad \widetilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\ ,\quad \tilde{p}_\mu = p_\mu - \frac{pk}{k^2}k_\mu\ ,\quad (ab)_{\mu\nu} = a_\mu b_\nu + a_\nu b_\mu\ ,$$

and quantities a_i on the right side of Eq. (5) are given in Appendix 1 of Ref. [9] (Preprint).

The constraints on the possible angles and energy fractions of positrons can be obtained from the condition on the invarinant mass of neutrinos: this last have to be positive

$$1 - y - x_1 - x_2 + \frac{x_1 y}{2}(1-c_1) + \frac{x_2 y}{2}(1-c_2) + \frac{x_1 x_2}{2}(1-c_{12}) > 0\ , \tag{6}$$

where $c_{12} = \cos\widehat{\vec{p}_1\vec{p}_2}$.

To simplify calculations we divide the positron energy fractions phase space by four kinematical regions, where restrictions on the positron energy fractions, positron angles and form of R on the right side of Eq. (3) are different. In the first region the condition

$$1 - x_1 - x_2 - y \geq 0$$

is satisfied. One can see from inequality (6) that in this case all angles for positrons are permitted. The quantity R in this region is very simple

$$R_1 = \frac{M^4}{2}\Big(\frac{1}{k^2 B} - \frac{1}{B^2}\Big)\ ,\quad B = (p_1+p_2+p_3)^2\ . \tag{7}$$

In the framework of chosen accuracy we can use $k^2+2p_2p_3$ for B in R_1 because the quantity p_1p_2 is always negligible in this region. Then the integration R_1 over the phase space of positrons gives well known result obtained in [8] (see also [9])

$$F_1(L,l)=\frac{1}{8}L^2+L(-1+\frac{1}{2}l)+\frac{1}{2}l^2-2l+3-\frac{\pi^2}{12}+O(1-y)\ . \tag{8}$$

In the second region the energy fractions of positrons satisfies inequalities

$$1-y-x_2<x_1<1-y,\ \ 0<x_2<1-y\ . \tag{9}$$

Because of the smallness both x_1 and x_2 in this region we can omit the last term in condition (6) and obtain the constraints on the c_1 and c_2 in the form

$$-1<c_2<1,\ \ -1<c_1<1+\frac{2(1-x_1-x_2-y)}{x_1y}\ , \tag{10}$$

and

$$-1<c_2<1+\frac{2(1-x_1-x_2-y)}{x_2y}+\frac{x_1}{x_2}(1-c_1)\ ,\ \ 1>c_1>1+\frac{2(1-x_1-x_2-y)}{x_1y}\ . \tag{11}$$

With the chosen accuracy the expression for R in the second region coincides with R_1. The angular integration of separate terms in R_1 over regions (10) and (11) gives

$$\int\limits_{(2)}dc_1dc_2\frac{M^4}{k^2B}=\frac{4}{x_1x_2}[-L\ln\frac{x_1+x_2+y-1}{yx_2}-Li_2(-\frac{x_1}{x_2})-\frac{1}{2}\ln^2\frac{x_1(x_1+x_2+y-1)}{y}$$

$$+2\ln x_1\ln x_2]\ ,\ \ \int\limits_{(2)}dc_1dc_2\frac{M^4}{B^2}=-\frac{4}{x_1x_2}\ln\frac{(x_1+x_2)(x_1+x_2+y-1)}{x_1x_2}\ , \tag{12}$$

where the form of B on the left side of relations (12) is the same as in the first kinematical region.

The further integration respect to energy fractions of the positrons defines the contribution of the second kinematical region (see inequalities (9)-(11)) into function $F(L,l)$

$$F_2(L,l)=\frac{1}{4}L+\frac{1}{2}l+\ln 2-2+\frac{\pi^2}{12}\ . \tag{13}$$

The restrictions on the energy fractions in the third region are as follows

$$0<x_1<1-y\ ,\ \ 1-y<x_2<1\ .$$

Note that the condition on the energies of the visible particles in decay (2) gives for the upper limit of x_2 the quantity $2-y-x_1$, which differs from 1 by value of the order $1-y$. The accounting of that difference leads to contribution of the order $1-y$ in $F(L,l)$ which is beyond our accuracy.

Let us analyze the condition (6) in the third kinematic region. If the values x_2 are near $(1-y)$ one can, as before, neglect the last term in (6) and obtain for c_2 the same constraints as in the first inequality in (11). The extension them on large values of $x_2\gg(1-y)$ means that corresponding values of c_2 are near -1 with $\Delta c_2\sim(1-y)$. The physical content of this circumstance is very transparent: in event with large–energy electron ($y\approx1$) the large energy

positron ($x_2 \gg 1-y$) must fly in the opposite direction respect to the direction of the electron 3–momentum. Contrary the conservation of 3–momentum in decay (2) would be violated.

Therfore, we can correct the above restriction taking into account the last term in (6) at $c_2 = -1$. This way we derive the constraints on the c_2 and c_1 in the third region

$$-1 < c_2 < -1 + \frac{2(1-y-x_1)(1-x_2)}{x_2 y} + \frac{x_1(y-x_2)}{x_2 y}(1-c_1) \ , \quad -1 < c_1 < 1 \ . \tag{14}$$

The quantity R in the third region reads

$$R_3 = \frac{M^4}{B^2}(-\frac{1}{2} + \frac{x_2}{2} + 2x_2^2) + \frac{M^2}{k^2}(-1 + 2x_2 + 6x_2^2) + \frac{a_{23}}{k^2}(-\frac{1}{2} - 7x_2) + \tag{15}$$

$$\frac{3a_{23}^2}{M^2 k^2} + \frac{M^4}{k^2 B}(\frac{1}{2} - x_2^2 - 2x_2^3) \ , \quad a_{23} = 2p_2 p_3 \ .$$

In accordance with our prescription in this region we have to take term $2(p_1 p_2)$, that enters in B, at $c_2 = -1$. Such procedure leads to

$$\frac{M^2}{B} = \frac{2}{x_2}(1 - c_2 + 2x_1 + \frac{x_1}{x_2}(y-x_2)(1-c_1))^{-1}$$

on the right side of Eq. (15).

The list of necessary angular integrals in R_3 reads

$$\int\limits_{(3)} dc_1 dc_2 \frac{M^4}{B^2} = \frac{4}{x_2 x_1}(\frac{x_1}{x_1+x_2+y-1} - \ln\frac{x_1+x_2}{x_2}) \ ,$$

$$\int\limits_{(3)} dc_1 dc_2 \frac{M^2}{k^2} = \int\limits_{(3)} \frac{dc_1 dc_2}{x_2}\frac{a_{23}}{k^2} = \int\limits_{(3)} \frac{dc_1 dc_2}{x_2^2}\frac{a_{23}^2}{k^2} = \frac{4(1-x_2)}{x_2 x_1}(x_1 + (1-y-x_1)(L + 2\ln x_1)) \ ,$$

$$\int\limits_{(3)} dc_1 dc_2 \frac{M^4}{k^2 B} = \frac{4}{x_1 x_2}[(L + 2\ln x_1)(\ln\frac{x_2(y+x_1)}{x_1+x_2+y-1} - x_2 \ln\frac{x_1+x_2}{x_2} - Li_2(-\frac{x_1}{x_2})] \ . \tag{16}$$

When writing these integrals we neglect all terms which lead to terms of the order $(1-y)^3$ in the electron spectrum.

Using Eqs. (16) and integrating over the posintron energy fractions we derive the contribution of the third kinematical region into function F(L,l) in the form

$$F_3(L,l) = -\frac{1}{3}L - \frac{1}{4}Ll - \frac{1}{2}l^2 - \frac{1}{6}l + \frac{31}{24} - \frac{\pi^2}{24} - \ln 2 \ . \tag{17}$$

In the fourth kinematical region

$$0 < x_2 < 1-y \ , \quad 1-y < x_1 < 1 \ .$$

The restrictions on the c_1 and c_2 in this region can be obtained from (14) by the simple substitution

$$x_1 \longleftrightarrow x_2 \ , \quad c_1 \longleftrightarrow c_2$$

because of obvious symmetry of the positron phase space in the third and fourth regions.

The expression for R in this region has the following form

$$R_4 = \frac{M^4}{B^2}(-\frac{1}{2}+x_1)(1+x_1) + \frac{M^4}{k^2B}(\frac{1}{2}-\frac{x_1}{2}-x_1^2-\frac{3x_1}{1+x_1}) + \frac{M^4}{k^4}\frac{x_1(1-2x_1)}{1+x_1} \ . \tag{18}$$

The angular integrals in considered case are

$$\int\limits_{(4)} dc_1dc_2\frac{M^4}{B^2} = \frac{4}{x_1x_2}(\frac{x_2}{x_1+x_2+y-1} - \ln\frac{x_1+x_2}{x_1}) \ ,$$

$$\int\limits_{(4)} dc_1dc_2\frac{M^4}{k^4} = \frac{4}{x_1^2}(-1+\ln\frac{x_1+x_2+y-1}{x_1+y-1}),$$

$$\int\limits_{(4)} dc_1dc_2\frac{M^4}{k^2B} = \frac{4}{x_1x_2}[Li_2(\frac{x_2}{x_1+x_2+y-1}) + Li_2(-\frac{x_2}{x_1}) -$$

$$Li_2(\frac{x_1x_2}{x_1+x_2+y-1}) - Li_2(-x_2)] \ . \tag{19}$$

Using expression for R_4 and angular integrals (19) we perform the integration respect to the positron energy fractions in fourth region and derive

$$F_4(L,l) = \frac{3}{4} - \ln 2 \ . \tag{20}$$

Thus, the spectrum of very fast electrons in five–lepton decay (2) (provided the energy fraction of one positron is smaller then $1-y$ and the energy fraction of other one is arbitrary) is defined as a sum of contributions of considered above four kinematical regions and can be written in the following form

$$\frac{d\Gamma}{\Gamma_0 d\,y} = \frac{\alpha^2}{\pi^2}(1-y)^2[\frac{1}{8}L^2 + (\frac{1}{4}l - \frac{13}{12})L - \frac{5}{3}l - \ln 2 - \frac{\pi^2}{24} + \frac{73}{24}] \ . \tag{21}$$

Note that terms containing L^2 and L in our final result coincide with those computed in [9], where collinear and semicollinear kinematics of the five–lepton μ decay has been investigated.

Here we considered the case when the energy of only one (from two) positron in decay (2) is small. But we sure that factor $(1-y)^2$ in the fast electron spectrum would appear at arbitrary positron energies because of essential squeeze of the angular phase space of the positrons along direction opposite to the electron 3–momentum, if their energies became large enough. The effect of such squeeze we observed in our analytical calculations. Therefore, we conclude that at study of the muonium–antimuionium conversion in vacuum by the observation of the very fast electron near its maximum energy, the probability of the main background decay always is very small. For example, in accordance with our estimations it consists for about $1.24\cdot 10^{-7}\Gamma_0$ if $y = 0.9$ and is on the essential decrease when the electron energy goes on to increase.

References

1. P. Herczeg and R.N. Mohapatra, Phys. Rev. Lett. **69**, 2475 (1992).

2. A.Halprin, Phys. Rev. Lett. **48**, 1313 (1982).

3. G.G. Wong and W.S. Hou, Phys. Rev. D **50**, R2962 (1994); W.S. Hou and G.G. Wong, Phys. Rev. D **53**, 1537 (1996).

4. A. Halprin and A. Masiero, Phys. Rev. D **48**, 2987, (1993).

5. H. Fujii, Y. Minura, K. Sasaki and T. Sasaki, Phys. Rev. D **49**, 559 (1994).

6. V.A. Gordeev et al., JETP Lett. **59**, 589 (1994); Yad. Fiz. **60**, 1291 (1997).

7. L. Williman et al., Phys. Rev. Lett. **82**, 49 (1999).

8. E.G. Drukarev et al., Preprint LNPI–1317 (1987); E.G. Drukarev and V.A. Gordeev, Preprint LNPI–1588 (1990).

9. A.B. Arbuzov, E.A. Kuraev, N.P. Merenkov, JINR Preprint E4–93–87, Dubna 1993; Pis'ma Zhur. Exp. Teor. Fiz. **57**, 746 (1993).

10. D.Yu. Bardin et al., JINR Preprint E2–5904, Dubna, 1971.